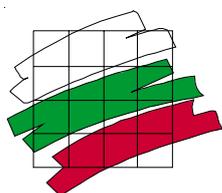


# WORKING PAPER SERIES

## EQUILIBRIUM EXCHANGE RATE DETERMINATION FOR THE CASE OF BULGARIA

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# 1 Introduction

The question of what is the “right” level of the exchange rate for a particular economy has received a lot of attention from economic researchers. Its importance for countries with fixed exchange rate regimes is especially prominent and economies running under currency board arrangements stand out as natural candidates for this type of analysis. Identifying exchange rate misalignments is important even if one does not have the policy option of a currency devaluation, since deviations from the equilibrium value of the exchange rate can have implications for future changes in competitiveness or output growth for the economy.

In principle, to tackle the calculation of an equilibrium exchange rate one has to have an equilibrium concept and a mechanism (i.e. a model) integrating different economic fundamentals into one coherent entity. Various approaches exist in the literature, as surveyed, for instance, by MacDonald (2000). Most of them rely on one or another measure of external equilibrium, although more sophisticated ones combine internal and external equilibrium to get a comprehensive picture of the state of the economy. Starting from simple absolute purchasing power parity (PPP) calculations, various extensions and modifications have been developed to model the equilibrium level of the exchange rate, e.g. fundamental equilibrium exchange rates, behavioral equilibrium exchange rates and capital enhanced equilibrium exchange rates. to name just a few. The modelling techniques used feature a diverse set of tools ranging from time series decompositions to cointegration analysis.

In this paper we tackle the issue of equilibrium exchange rate determination in the case of Bulgaria. We develop a simple dynamic model for calculating the equilibrium path of the nominal exchange rate over a specified time horizon. Bearing in mind the specific situation of Bulgaria as a country with a currency board, we model the lev-Euro (BGN-EUR) exchange rate, taking the US dollar-Euro (USD-EUR) exchange rate as given. In our setup the country’s trading partners are divided into two groups: USD-trading and EUR-trading countries, the latter considered equivalent to the European Union (EU) and the former assumed to comprise the non-EU trading partners of Bulgaria. Our concept of equilibrium requires the current account of the balance of payments to converge to zero over time, while at the same time achieving a certain degree of price convergence with the EU. With these requirements imposed exogenously, we calculate the BGN-EUR exchange rate trajectory that satisfies them under the model. To attain sufficient proximity to the actual characteristics of the Bulgarian economy, we

calibrate the model with post currency board data for the country prior to performing the simulations. The details are supplied below.

The remainder of the paper is organized as follows. Section 2 develops the model we use. Section 3 describes the estimation strategy and spells out the assumptions used to complete the specification. Section 4 details the simulation results and section 5 analyzes the sensitivity of our results with respect to the assumptions. Section 6 concludes.

## 2 The Model

Our approach relies on a setup using the concept of real current account (Obstfeld and Rogoff, 1996, p.18). It should be borne in mind, therefore, that the calculations would differ to a certain extent from projections of the nominal figures for the respective aggregates. Without explicitly specifying it further, we adopt the convention that the values of the quantities used below are known for a given base year,  $t = 0$ . The latter assumption is easily justified by the fact that simulations based on the model are performed using actual data for the starting year.

Let the real growth rate of EUR-denominated exports of goods and services be determined by the equation

$$\frac{dX_t^\epsilon}{X_t^\epsilon} = \alpha_\epsilon \left( \frac{dS_t^\epsilon}{S_t^\epsilon} + \frac{dP_t^{\epsilon,*}}{P_t^{\epsilon,*}} - \frac{dP_t}{P_t} \right) + \beta_\epsilon \frac{dY_t^{\epsilon,*}}{Y_t^{\epsilon,*}};$$

or, in finite-difference terms,

$$\frac{\Delta X_t^\epsilon}{X_{t-1}^\epsilon} = \alpha_\epsilon \left( \frac{\Delta S_t^\epsilon}{S_{t-1}^\epsilon} + \frac{\Delta P_t^{\epsilon,*}}{P_{t-1}^{\epsilon,*}} - \frac{\Delta P_t}{P_{t-1}} \right) + \beta_\epsilon \frac{\Delta Y_t^{\epsilon,*}}{Y_{t-1}^{\epsilon,*}}, \quad (1)$$

where  $X_t^\epsilon$  denotes period  $t$  real exports to partner countries which trade in EUR ( $\epsilon$  stands for EUR throughout the exposition),  $P_t$  denotes the price level in the home country and  $P_t^{\epsilon,*}$  and  $Y_t^{\epsilon,*}$  stand for the price level and income in EUR-trading partner countries, respectively.  $S_t^\epsilon$  is the exchange rate of the local currency to the EUR expressed as local currency per 1 EUR. As usual, asterisks are used to denote “foreign” variables.  $\alpha_\epsilon$  and  $\beta_\epsilon$  are the elasticities of exports with respect to the real exchange rate and foreign income, correspondingly.

Analogously, USD-denominated exports growth in real terms is given by

$$\frac{\Delta X_t^\$}{X_{t-1}^\$} = \alpha_\$ \left( \frac{\Delta S_t^\$}{S_{t-1}^\$} + \frac{\Delta P_t^{\$,*}}{P_{t-1}^{\$,*}} - \frac{\Delta P_t}{P_{t-1}} \right) + \beta_\$ \frac{\Delta Y_t^{\$,*}}{Y_{t-1}^{\$,*}}, \quad (2)$$

with the meaning of the notation preserved.

Real imports of goods and services are described by

$$\frac{\Delta M_t^\epsilon}{M_{t-1}^\epsilon} = \gamma_\epsilon \left( \frac{\Delta S_t^\epsilon}{S_{t-1}^\epsilon} + \frac{\Delta P_t^{\epsilon,*}}{P_{t-1}^{\epsilon,*}} - \frac{\Delta P_t}{P_{t-1}} \right) + \delta_\epsilon \frac{\Delta Y_t}{Y_{t-1}}, \quad (3)$$

for EUR-denominated trade and by

$$\frac{\Delta M_t^\$}{M_{t-1}^\$} = \gamma_\$ \left( \frac{\Delta S_t^\$}{S_{t-1}^\$} + \frac{\Delta P_t^{\$,*}}{P_{t-1}^{\$,*}} - \frac{\Delta P_t}{P_{t-1}} \right) + \delta_\$ \frac{\Delta Y_t}{Y_{t-1}}, \quad (4)$$

for USD-denominated trade, where  $Y_t$  stands for domestic income and  $\gamma$  and  $\delta$  are again elasticities.

Total exports are given by

$$X_t = X_t^\epsilon + X_t^\$,$$

so that

$$\frac{\Delta X_t}{X_{t-1}} = \frac{\Delta X_t^\epsilon}{X_{t-1}^\epsilon} \cdot \frac{X_{t-1}^\epsilon}{X_{t-1}} + \frac{\Delta X_t^\$}{X_{t-1}^\$} \cdot \frac{X_{t-1}^\$}{X_{t-1}} = w_{X,t-1}^\epsilon \cdot \frac{\Delta X_t^\epsilon}{X_{t-1}^\epsilon} + w_{X,t-1}^\$ \cdot \frac{\Delta X_t^\$}{X_{t-1}^\$}. \quad (5)$$

Similarly, for total imports

$$M_t = M_t^\epsilon + M_t^\$,$$

and

$$\frac{\Delta M_t}{M_{t-1}} = \frac{\Delta M_t^\epsilon}{M_{t-1}^\epsilon} \cdot \frac{M_{t-1}^\epsilon}{M_{t-1}} + \frac{\Delta M_t^\$}{M_{t-1}^\$} \cdot \frac{M_{t-1}^\$}{M_{t-1}} = w_{M,t-1}^\epsilon \cdot \frac{\Delta M_t^\epsilon}{M_{t-1}^\epsilon} + w_{M,t-1}^\$ \cdot \frac{\Delta M_t^\$}{M_{t-1}^\$}. \quad (6)$$

We assume that the economy under consideration satisfies a small country assumption in the sense that it cannot influence the exchange rate USD/EUR (USD per 1 EUR), denoted here by  $S_t^{\$/\epsilon}$ . This is a natural assumption in the case of Bulgaria, as the latter's economy is running under a currency board

arrangement and is obviously too small to affect the USD/EUR exchange rate. Then a no-arbitrage argument shows that

$$S_t^{\$} = \frac{S_t^{\epsilon}}{S_t^{\$/\epsilon}}. \quad (7)$$

By definition, the current account for period  $t$  is

$$CA_t \equiv X_t - M_t + Yf_t + Tr_t, \quad (8)$$

with  $Yf_t$  and  $Tr_t$  denoting net factor income from abroad and net transfers from abroad, respectively. In our case, we can take  $Yf_t$  and  $Tr_t$  as known in advance for every period  $t$ , since they are determined primarily by exogenous factors – international interest rates and EU pre-accession funds, respectively. Then we can define

$$K_t \equiv Yf_t + Tr_t$$

and rewrite the current account identity as

$$CA_t \equiv X_t - M_t + K_t. \quad (9)$$

Since foreign incomes and prices are determined outside the domestic economy, they can be treated as exogenous to the model, together with the USD/EUR exchange rate. Then the current account becomes a function of  $S_t^{\epsilon}$ ,  $Y_t$  and  $P_t$ , or their growth rates, assuming that all the relevant variables are known in levels for an initial period  $t = 0$ .

Let us further introduce the following model for  $Y_t$ :

$$\frac{\Delta Y_t}{Y_{t-1}} = \frac{\Delta \bar{Y}_t}{\bar{Y}_{t-1}} + \kappa \left( \left( \frac{\Delta P_t}{P_{t-1}} - \frac{\Delta P_t^{\mathbf{E}}}{P_{t-1}^{\mathbf{E}}} \right) - \frac{1}{2\pi_{gap}} \left( \frac{\Delta P_t}{P_{t-1}} - \frac{\Delta P_t^{\mathbf{E}}}{P_{t-1}^{\mathbf{E}}} \right)^2 \right) \quad (10)$$

where  $\kappa > 0$ ,  $\frac{\Delta \bar{Y}_t}{\bar{Y}_{t-1}}$  is the growth rate of potential output (exogenously given) and  $\frac{\Delta P_t^{\mathbf{E}}}{P_{t-1}^{\mathbf{E}}}$  is expected inflation. This gives the highest rate of growth when  $\left( \frac{\Delta P_t}{P_{t-1}} - \frac{\Delta P_t^{\mathbf{E}}}{P_{t-1}^{\mathbf{E}}} \right) = \pi_{gap}$ , i.e. the optimal gap between actual and expected inflation.

In order to dispense with cumbersome notation, in the following we will use the symbol  $\vartheta$  with a subscript to denote a percentage change in a variable. Thus,  $\vartheta_{P_t^{\mathbf{E}}}$  will mean the percentage change in the expected price level, or expected inflation. Therefore, (10) can equivalently be rewritten as

$$\vartheta_{Y_t} = \vartheta_{\bar{Y}_t} + \kappa \left[ \left( \vartheta_{P_t} - \vartheta_{P_t^{\mathbf{E}}} \right) - \frac{1}{2\pi_{gap}} \left( \vartheta_{P_t} - \vartheta_{P_t^{\mathbf{E}}} \right)^2 \right], \quad \kappa > 0. \quad (11)$$

For the expectations component in (11) we postulate the following expectation formation scheme:

$$\vartheta_{P_t^E} = \eta \vartheta_{P_{t-1}} + (1 - \eta) \vartheta_{P_t^{\epsilon,*}}, \quad 0 < \eta < 1. \quad (12)$$

Although various choices of expectation formation mechanisms can go in this equation, we chose this one due to the fact that in our opinion it successfully represents the typical way perceptions about prices are formed in transition economies similar to Bulgaria – as a mixture of historical experience and objective (external) factors.

Equation (9) implies that

$$\vartheta_{CA_t} = \vartheta_{X_t} \frac{X_{t-1}}{CA_{t-1}} - \vartheta_{M_t} \frac{M_{t-1}}{CA_{t-1}} + \vartheta_{K_t} \frac{K_{t-1}}{CA_{t-1}}. \quad (13)$$

Then,

$$\begin{aligned} \vartheta_{CA_t} &= (w_{X,t-1}^\epsilon \vartheta_{X_t^\epsilon} + w_{X,t-1}^\$ \vartheta_{X_t^\$}) \frac{X_{t-1}}{CA_{t-1}} \\ &- (w_{M,t-1}^\epsilon \vartheta_{M_t^\epsilon} + w_{M,t-1}^\$ \vartheta_{M_t^\$}) \frac{M_{t-1}}{CA_{t-1}} + \vartheta_{K_t} \frac{K_{t-1}}{CA_{t-1}}. \end{aligned} \quad (14)$$

Substituting the expressions for  $\vartheta_{X_t^\epsilon}$ ,  $\vartheta_{X_t^\$}$ ,  $\vartheta_{M_t^\epsilon}$  and  $\vartheta_{M_t^\$}$  in (14), we obtain

$$\begin{aligned} \vartheta_{CA_t} &= \left\{ w_{X,t-1}^\epsilon \left[ \alpha_\epsilon (\vartheta_{S_t^\epsilon} + \vartheta_{P_t^{\epsilon,*}} - \vartheta_{P_t}) + \beta_\epsilon \vartheta_{Y_t^{\epsilon,*}} \right] \right. \\ &\quad \left. + w_{X,t-1}^\$ \left[ \alpha_\$ (\vartheta_{S_t^\$} + \vartheta_{P_t^{\$,*}} - \vartheta_{P_t}) + \beta_\$ \vartheta_{Y_t^{\$,*}} \right] \right\} \frac{X_{t-1}}{CA_{t-1}} \\ &- \left\{ w_{M,t-1}^\epsilon \left[ \gamma_\epsilon (\vartheta_{S_t^\epsilon} + \vartheta_{P_t^{\epsilon,*}} - \vartheta_{P_t}) + \delta_\epsilon \vartheta_{Y_t} \right] \right. \\ &\quad \left. + w_{M,t-1}^\$ \left[ \gamma_\$ (\vartheta_{S_t^\$} + \vartheta_{P_t^{\$,*}} - \vartheta_{P_t}) + \delta_\$ \vartheta_{Y_t} \right] \right\} \frac{M_{t-1}}{CA_{t-1}} \\ &+ \vartheta_{K_t} \frac{K_{t-1}}{CA_{t-1}}. \end{aligned} \quad (15)$$

We are interested in finding a sequence of exchange rates  $S_t^\epsilon$  over a specified finite time horizon  $t = 1, \dots, T$  such that the current account is contained within desirable bounds. In particular, we constrain the current account to follow a process of the form:

$$CA_{t+i} = \rho CA_t, \quad \forall t \text{ with } \rho \in (0, 1) \text{ and } i > 0. \quad (16)$$

To simplify the interpretation of the results, we work with  $i = 1$  throughout the paper.

From (16) it follows that

$$\begin{aligned}\Delta CA_{t+i} &= \rho \Delta CA_t; \\ \frac{\Delta CA_{t+i}}{CA_{t+i-1}} \cdot \frac{CA_{t+i-1}}{CA_{t-1}} &= \rho \frac{\Delta CA_t}{CA_{t-1}}; \\ \vartheta_{CA_{t+i}} \cdot \frac{CA_{(t-1)+i}}{CA_{t-1}} &= \rho \vartheta_{CA_t}\end{aligned}$$

or, ultimately,

$$\vartheta_{CA_{t+i}} = \vartheta_{CA_t}. \quad (17)$$

To obtain a well-determined system we also need to impose an additional condition on the dynamics of  $P_t$ . Our approach is to require

$$P_T = \tau P_T^{\epsilon,*}, \quad \tau > 0, \quad (18)$$

the intuition being that we would like to obtain some degree of price level convergence as well as a balanced current account at the end of the specified horizon.

### 3 Estimation and Simulation Strategy

The coefficients in equations (1) through (4) are estimated from quarterly data for the period Q1:1998 - Q1:2002. We choose this period to avoid the issue of structural breaks, doing so at the cost of having a small sample size at our disposal. The data on Bulgaria's GDP deflator and real growth, as well as on real exports and imports of goods and services, were taken from national accounts data provided by the Bulgarian statistical office (the National Statistical Institute). Data on the growth rates and deflators of partner countries were provided by the International Monetary Fund. More specifically, the growth rate and deflator for the EUR-trading countries were proxied by the respective series for the Euro area and the corresponding aggregates for the USD-trading countries were proxied by a combination of the data for the USA and Japan, weighted by 5/6 and 1/6 respectively. The data on the required exchange rates were taken from Bulgarian National Bank publications. It must be noted that the estimation of the coefficients was carried out for the purpose of providing initial values for the simulations

in the next section, therefore the reported estimates should not be regarded as the outcome of a full-fledged econometric exercise.

The estimation approach we employ is a Bayesian regression one (see e.g. Hamilton, 1994, chap. 12 and the references therein)<sup>1</sup>. Briefly, the Bayesian approach allows for coefficient estimation that integrates the sample information with prior (extraneous to the estimation problem) knowledge about the phenomenon being modelled. If we denote by  $\mathbf{x} = (x_1, \dots, x_n)$  a sample of  $n$  (possibly vector-valued) observations available and by  $\theta \in \mathbb{R}^m$  the vector of coefficients to be estimated, then the likelihood function of the model can be written as  $p(\mathbf{x}|\theta)$ . In a Bayesian framework, the coefficients are treated as random variables and can correspondingly be described by their joint distribution. The prior information available is captured by the prior distribution of the coefficient vector,  $p(\theta)$ . Then one can use Bayes' formula to update prior knowledge in the light of the sample information available, which is formally specified through the conditional distribution of  $\theta$  given  $\mathbf{x}$ :

$$p(\theta|\mathbf{x}) = \frac{p(\theta)p(\mathbf{x}|\theta)}{p(\mathbf{x})} \text{ or, } p(\theta|\mathbf{x}) \propto p(\theta)p(\mathbf{x}|\theta).$$

The advantages of working in a Bayesian framework come from two directions. First, having prior knowledge at one's disposal helps alleviate problems associated with small sample size, since the researcher starts from an allegedly informed position and is thus able to reach certain conclusions on the basis of a less informative dataset. Second, the Bayesian approach allows us to sidestep the issue of shocks within the sample period as the prior naturally gives higher weight to outcomes which are deemed more plausible. These advantages come at a certain cost, however. Intuitively, strong prior beliefs can mislead one into disregarding important conclusions conveyed by the data if the latter contradict the researcher's prior expectations. Worse still, in extreme cases Bayesian analysis can be abusively employed to produce the desired result, practically ignoring the sample evidence (think of the exceptional case where probability one is assigned to a given outcome in the parameter space). Therefore to ensure certain analytical objectivity one should avoid excessively strong prior specifications<sup>2</sup> and, as a minimal requirement, clearly specify the prior distribution used in the analysis to enable others to judge whether the prior leads to distorted conclusions.

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<sup>1</sup>Computations reported in this section were undertaken using the Bayesian Analysis, Computation and Communication (BACC) software (<http://www.econ.umn.edu/~bacc>) described in Geweke (1999).

<sup>2</sup>This, of course, is a subjective issue in its own right.

Usually the resulting conditional distribution  $p(\theta|\mathbf{x})$ , known as the posterior distribution, is analytically intractable and it is therefore more convenient to simulate a sample from the posterior and investigate its properties. To this end, Markov chain Monte Carlo (MCMC) techniques are used to ensure convenient simulation even when the posterior is known only up to a factor of proportionality (see Tierney, 1994). Once the posterior has been obtained, various summary statistics can be used to describe the distribution and produce point estimates of  $\theta$ . The latter is typically done in a decision-theoretic framework, with an explicit loss function specified. For instance, a quadratic loss function implies the mean of the distribution as the optimal point estimate.

In the regression case, if  $\mathbf{y}$  is the vector of observations on the variable of interest and  $\mathbf{X}$  is the matrix of observations on the explanatory variables, the regression model can be written as

$$\mathbf{y} = \mathbf{X}\beta + \epsilon,$$

where  $\beta$  is the vector of coefficients to be estimated and  $\epsilon$  is the vector of disturbances (usually assumed to be independent and identically normally distributed). In the case of a model with unknown variance  $\sigma^2$  and on the assumption of normal disturbances the sample likelihood function for sample size  $T$  may be written as

$$p(\mathbf{y}|\beta, \sigma^2, \mathbf{X}) = \frac{1}{(2\pi\sigma^2)^{T/2}} \exp \left\{ \left[ -\frac{1}{2\sigma^2} \right] (\mathbf{y} - \mathbf{X}\beta)'(\mathbf{y} - \mathbf{X}\beta) \right\}$$

and appropriate priors may be assumed for  $\beta$  and  $\sigma^2$ .

To estimate the coefficients of equations (1) to (4) we use the normal linear model module of BACC in a single equation framework. The coefficient vector  $\beta$  is assigned a normal prior and the model works with a Wishart-distributed precision parameter  $H$  (the inverse of the variance) instead of the usual variance. When estimating a single equation, the precision parameter  $H$  is a scalar.  $H$  and  $\beta$  are assumed independent and the marginal distributions for the priors are

$$\beta \sim N(\underline{\beta}, \underline{H}_\beta^{-1}) \tag{19}$$

$$H \sim Wi(\underline{S}^{-1}, v), \tag{20}$$

where the vector  $\underline{\beta}$ , the matrix  $\underline{H}_\beta^{-1}$ , and the scalars  $\underline{S}^{-1}$  and  $v$  are specified by the researcher to reflect prior information/beliefs. For further details on

Table 1: Parameters of the prior distributions of equations (1) - (4)

<b>Equations (1) and (2)</b>	
$\underline{\beta}$	$(1, 3)'$
$\underline{H}_\beta$	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$
$\underline{v}$	1
$\underline{S}$	1
<b>Equations (3) and (4)</b>	
$\underline{\beta}$	$(-0.5, 3)'$
$\underline{H}_\beta$	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$
$\underline{v}$	1
$\underline{S}$	1

Table 2: Posterior moments of the parameters of equations (1) - (4)

	$\alpha_\epsilon$	$\beta_\epsilon$	$h_1$	$\alpha_\$$	$\beta_\$$	$h_2$	$\gamma_\epsilon$	$\delta_\epsilon$	$h_3$	$\gamma_\$$	$\delta_\$$	$h_4$
Mean	0.138	3.236	0.006	1.252	2.292	0.008	-1.004	2.579	0.008	0.620	2.332	0.008
Variance	0.077	0.302	0.000	0.054	0.275	0.000	0.066	0.183	0.000	0.056	0.150	0.000

the model and its computational aspects the reader is referred to McCausland and Stevens (2001). Table 1 lists the prior specifications used for the parameters of the respective equations. As mentioned above, the choice of priors is unashamedly subjective and can be construed to reflect our best personal knowledge of the phenomena under investigation.

For the above prior specifications and data we simulated a sample of 10000 observations from the posterior distribution using the built-in Gibbs sampler of the BACC software. To allow for a sufficient burn-in period the first 2000 observations were discarded. For the truncated sample thus obtained we calculated the mean and the variance as a (crude) summary of its features<sup>3</sup>. The results are reported in Table 2. The means of the respective parameters were then used as point estimates of the true values.

Using the above results, we produce simulations from the model for the

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<sup>3</sup>We also repeated the exercise on the truncated sample by taking every third observation to reduce autocorrelation. The results, however, differ insignificantly from the baseline calculation, with differences of the order of 0.001-0.002.

period 2002-2019. The essence of our computations lies in solving for the exchange rate that satisfies the current account and price convergence constraints (17) and (18). The model, however, is sufficiently flexible to be turned around in order to produce answers to other questions, for instance, what would the current account path be for a given set of external environment variables and for a given (possibly constant) exchange rate trajectory. We implement versions of the above exercises in the next section.

As for the data and parameters used, the projections for inflation and output growth in Bulgaria, the Eurozone and the rest of the world are taken from IMF forecasts. Income and current transfers are taken to increase gradually from 175 to 515 over the simulation period, consistent with expectations about their paths conditional on the other exogenous variables in the model. The price dynamics used imply only a modest degree of price level convergence, with  $\tau$  standing at about 0.54 at the end of the period. We assume parameters  $\eta = 0.5$ ,  $\kappa = 1$ ,  $\rho = 0.8$  and  $\pi_{gap} = 0.06$  in what follows but later provide results from sensitivity analysis conducted to test the robustness of our conclusions to changes in the assumptions. Details of the calculations and the data used are available from the authors.

## 4 Results from the model

In the base case, using the trade elasticities estimated by Bayesian analysis, and taking these elasticities as constant over time, we can consider two scenarios:

1. If the BGN-EUR exchange rate remains fixed, what will happen to the current account?
2. If the current account deficit is to decline by a fixed proportion each year, what should happen to the exchange rate?

The first scenario shows a highly unsustainable - in fact, rapidly rising - current account deficit (Figure 1).

The second scenario (Figure 2) shows that a very high depreciation of the BGN-EUR rate would be necessary to achieve a satisfactory current account deficit.

These results are explicable from the model's specification and the forecasts used for growth and inflation. In the medium term, i.e. after ap-

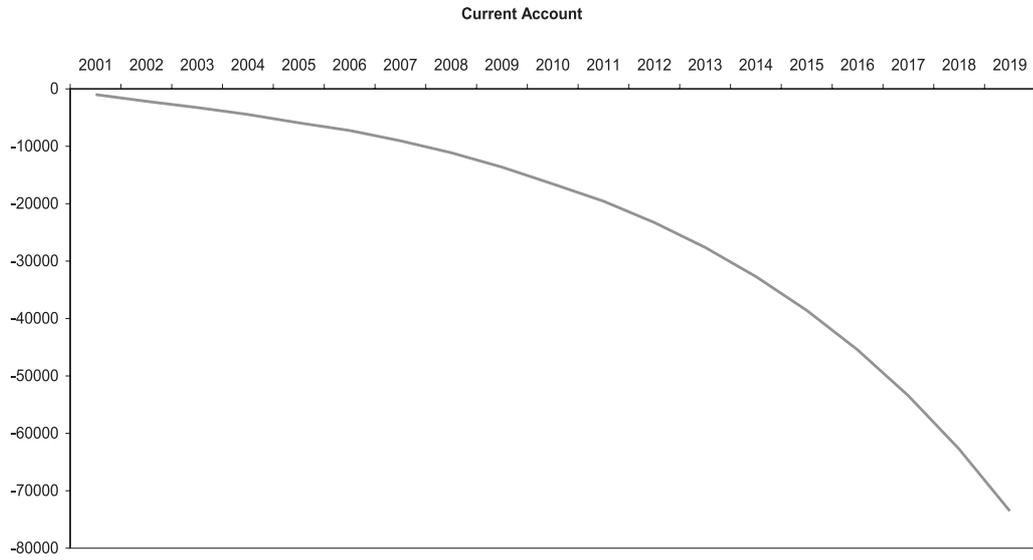


Figure 1: Current account simulation for the first scenario.

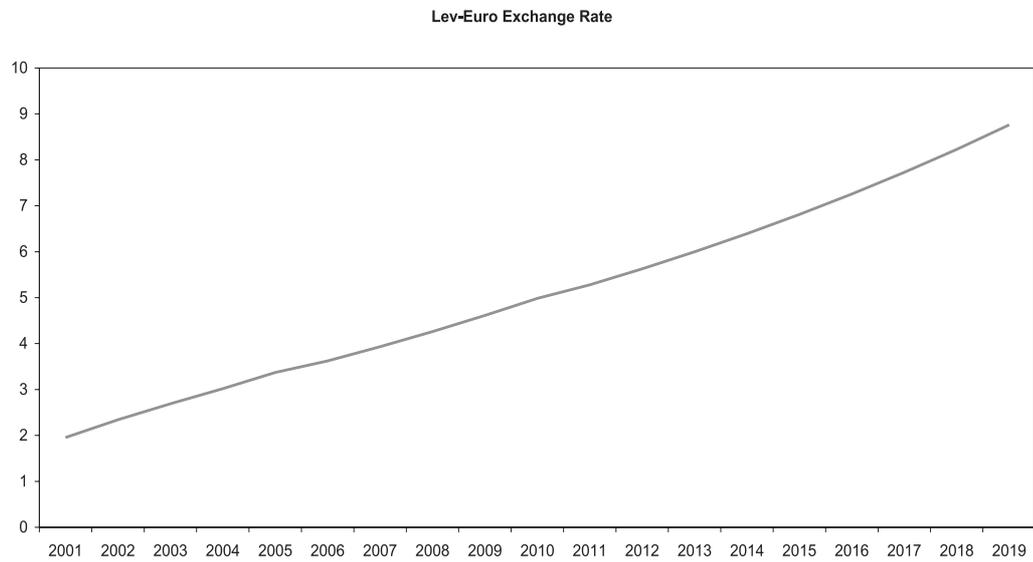


Figure 2: Exchange rate path required to observe current account dynamics.

proximately five years, the forecasts for output growth and inflation take on constant values. This is true for all regions. Since the growth rates of exports and imports in this model are determined by output growth, inflation and the change in the exchange rate (equations (1)-(4)), in the medium term this model either predicts constant growth for imports and exports (if the exchange rate is fixed), or implies a need for rapid depreciation of the exchange rate (in order to achieve a sustainable current account balance). Given the results from the first scenario, it is clear that in the absence of exchange rate flexibility this model predicts stronger growth of imports than exports, causing the current account deficit to explode.

One possible adjustment to the model is to make the trade elasticities vary over time. Given Bulgaria's position as a transition economy, it seems likely that its trade will become less sensitive as time goes by. We modify the trade equations such that the elasticities start from the values estimated by Bayesian analysis, and then decline exponentially over time, e.g.:

$$\alpha_{\epsilon,t} = \alpha_{\epsilon,0} \exp(-\lambda_{\epsilon}^{\alpha} t) \quad (21)$$

and similarly for the other coefficients, where  $\alpha_{\epsilon,0}$  is the original value and  $\lambda_{\epsilon}^{\alpha}$  is a time trend.

It seems plausible that the elasticities of imports should decline faster than those of exports. In what follows we assume that  $\lambda_{\epsilon}^{\alpha} = \lambda_{\S}^{\alpha} = \lambda_{\epsilon}^{\beta} = \lambda_{\S}^{\beta} = 0.1$ , while  $\lambda_{\epsilon}^{\gamma} = \lambda_{\S}^{\gamma} = \lambda_{\epsilon}^{\delta} = \lambda_{\S}^{\delta} = 0.2$ . We repeat our questions from above: what happens to the current account balance if the exchange rate remains fixed? What should happen to the exchange rate in order to obtain a declining current account deficit?

The first scenario shows a current account deficit that initially rises and then begins to fall (see Figure 3).

The second scenario, shown in Figure 4, requires that the exchange rate should first depreciate, peak in 6-7 years after the initial period and then experience a gradual appreciation.

According to these results, the Bulgarian economy will not be able to support an improving current account in the immediate future without either a depreciating exchange rate, or a change in economic fundamentals (e.g. potential output growth). If the currency board arrangement is maintained with the exchange rate at its present level, the current account will deteriorate further, only beginning to recover after 7-8 years have elapsed after the beginning of the simulation, i.e. after 2008 in our case.

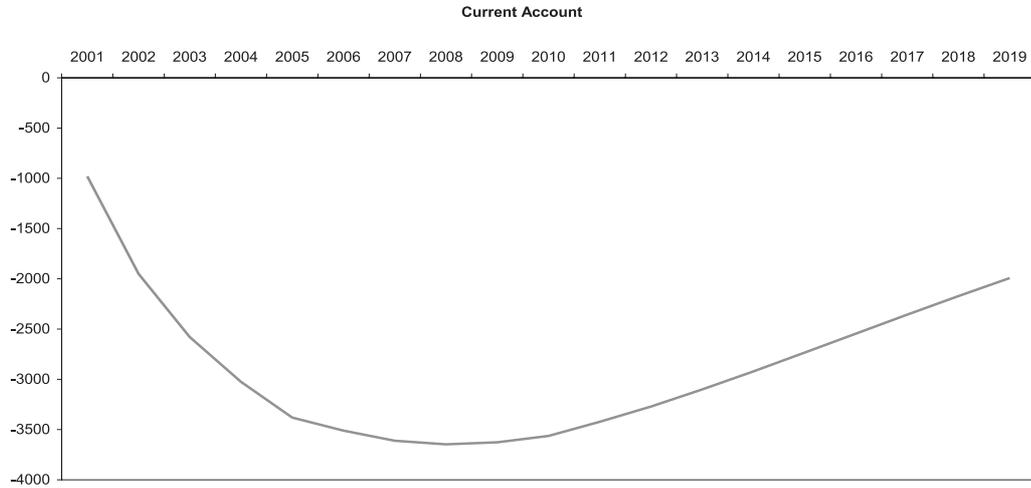


Figure 3: Current account simulation for the first scenario with changing elasticities.

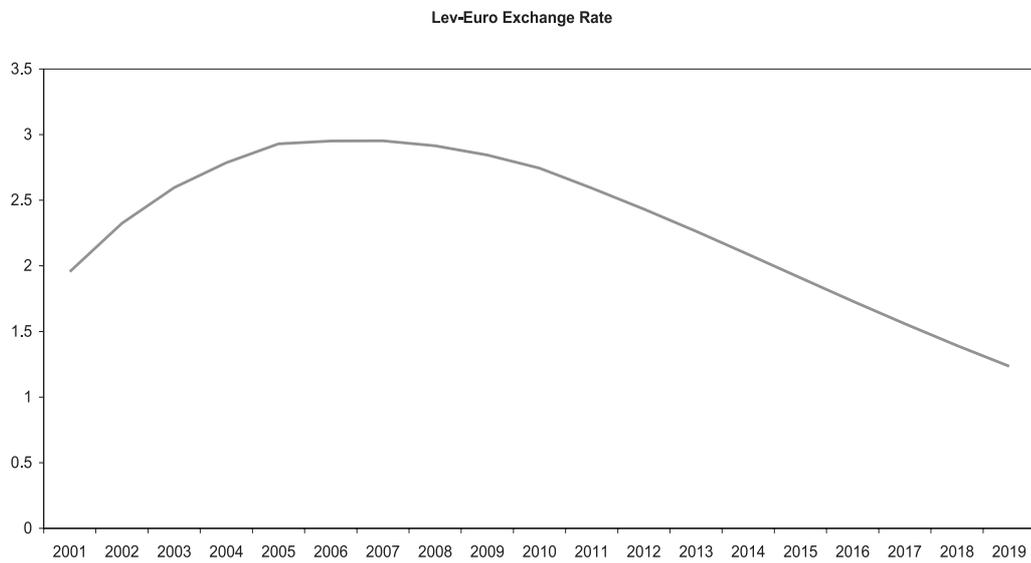


Figure 4: Exchange rate trajectory for the second scenario with changing elasticities.

## 5 Sensitivity Analysis

In this section we discuss the sensitivity of the results with respect to the assumptions. Considering the trade equations, it is clear that testing the sensitivity of the results to changes in  $\lambda_\epsilon^\beta$ , for example, is equivalent to testing for changes in  $\vartheta_{Y_t^{\epsilon,*}}$ . If the export elasticity declines at a slower rate, this has the same effect as if growth in European output is higher. Thus it is sufficient to check what happens when the time trends etc. are altered, without also checking the effect of varying predictions for growth, inflation, and so on. We also examine the effect of varying the parameters  $\eta, \kappa, \rho$  and  $\pi_{gap}$ . For brevity, the results of the sensitivity analysis are only reported for the scenario in which the current account is imposed, and the exchange rate allowed to vary (Figures 5 - 8):

As the charts show, the results of this model are absolutely insensitive to the choice of parameters  $\rho$  and  $\pi_{gap}$ . Varying the parameters  $\eta$  and  $\kappa$  changes the extent of the depreciation necessary (the peaks vary between 2.8 and 3.1 BGN/EUR), but the time of the peak and the general trajectory remain quite unchanged. In order to achieve the desired current account balance, the BGN should depreciate for the first few iterations of the model, say until 2006/7, after which it may appreciate somewhat. It should explicitly be stated that the model does not incorporate in any way possible effects from changes in trade regimes, decreased uncertainty in the economy, declining risk premia etc. hence the influence of joining an integration bloc such as the EU has been left out completely.

The more interesting outcome is from sensitivity tests on the trade elasticity trends. The results in the previous section are not at all sensitive to changes in the  $\lambda^\alpha$  or  $\lambda^\gamma$  parameters. However the elasticities with respect to income, which depend on parameters  $\lambda^\beta$  and  $\lambda^\delta$ , are influential. Figure 7 shows a paradoxical situation in which Bulgaria might become over-competitive and require a large exchange rate appreciation, due to the persistent strength of exports, i.e. low  $\lambda^\beta$ . This is not a situation that causes us serious concern. In the opposite scenario, i.e. high  $\lambda^\beta$ , the results are less sensitive, although weaker export growth unsurprisingly causes a weaker final value of the BGN.

Finally, the parameter  $\lambda^\delta$  is critical to the exchange rate trajectory. As Figure 8 illustrates, if the sensitivity of growth in imports with respect to domestic output growth does not decline sufficiently quickly, the exchange rate may be required to undergo a continuous depreciation, without ever reaching

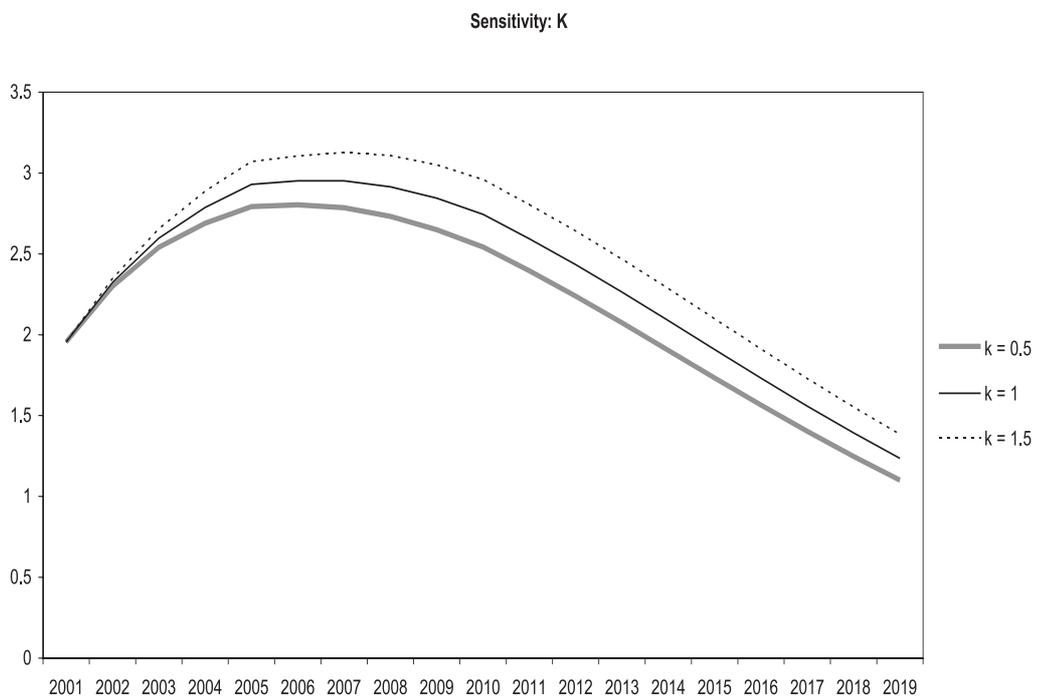
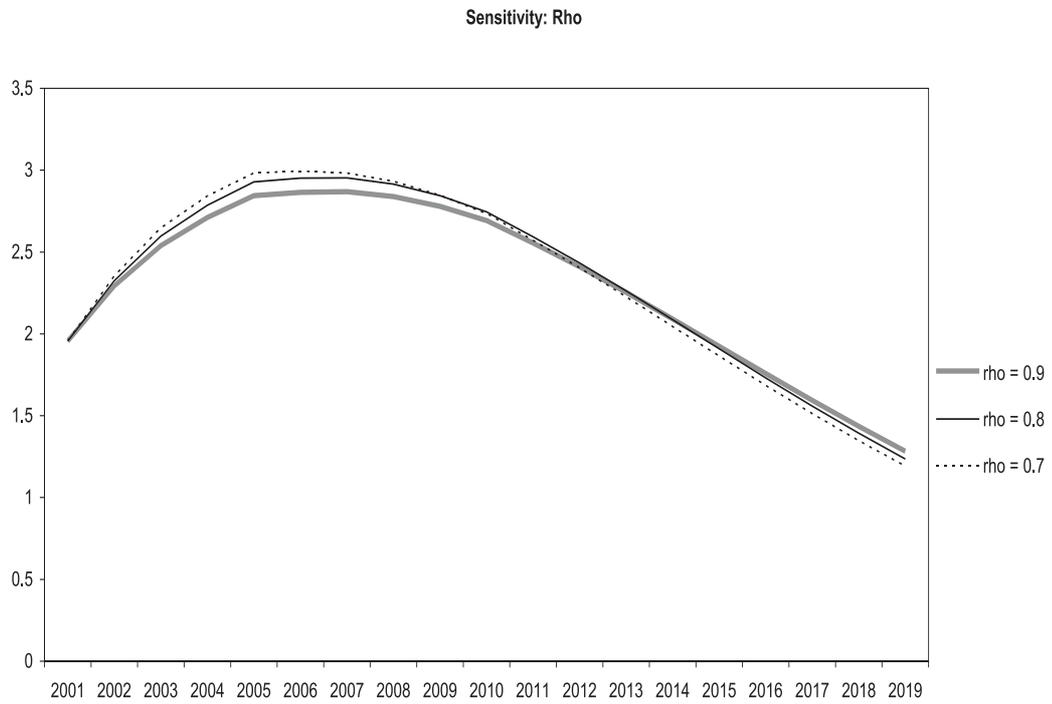


Figure 5: Sensitivities for  $\rho$  and  $\kappa$ .

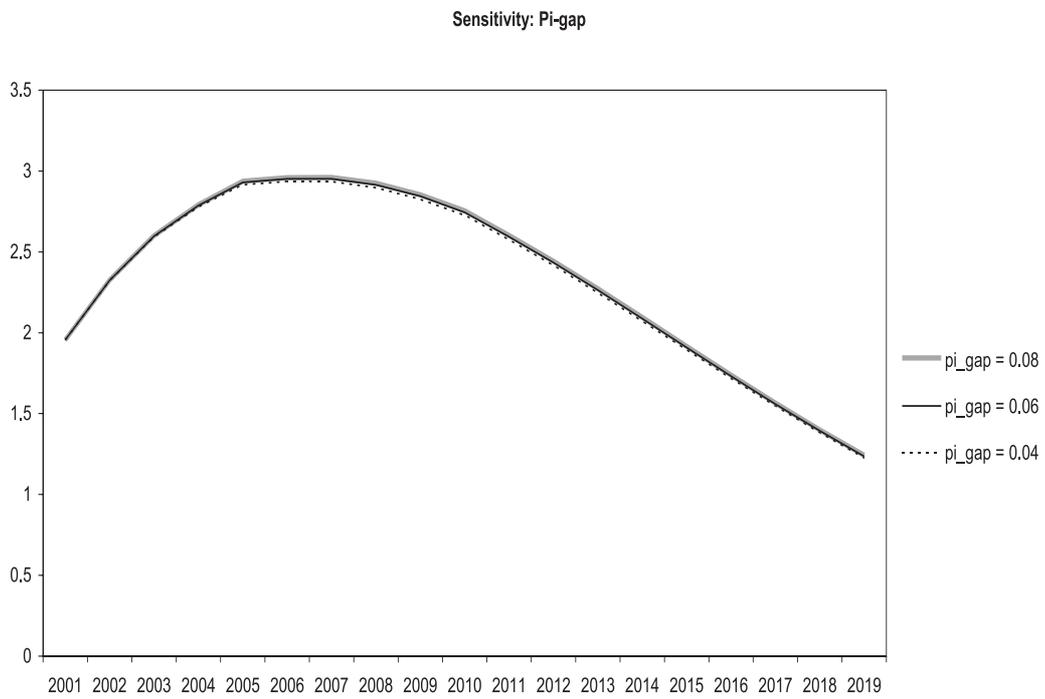
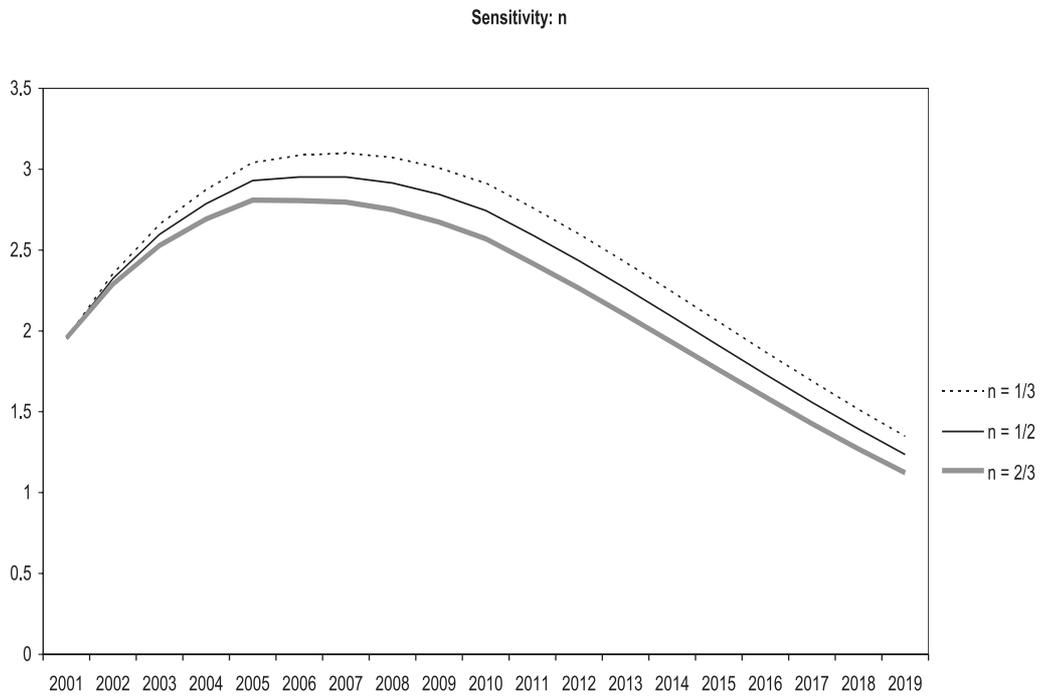
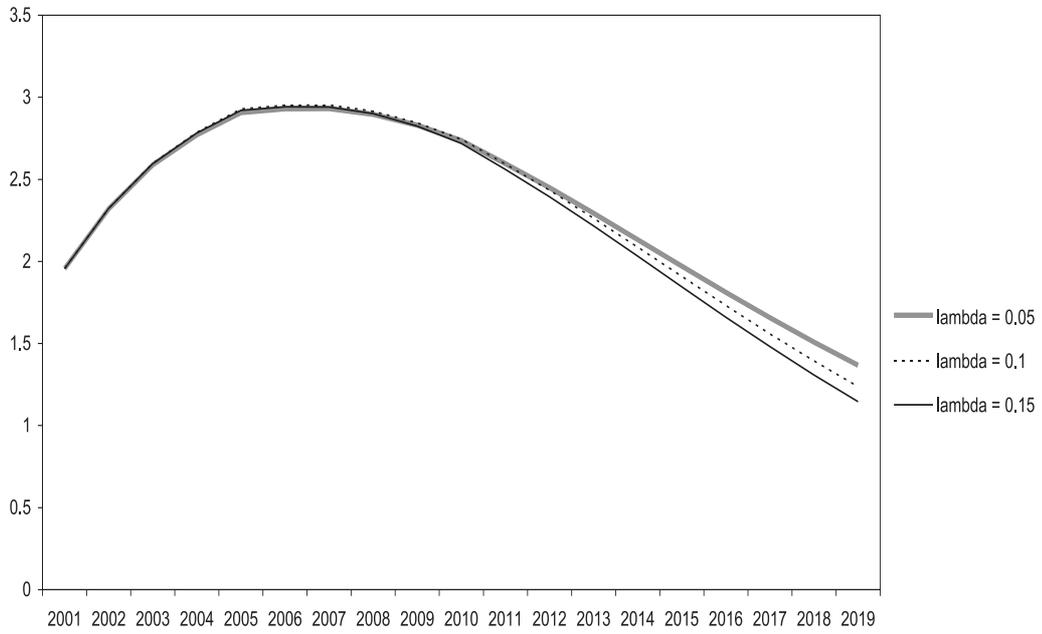


Figure 6: Sensitivities for  $\eta$  and  $\pi_{gap}$ .

Sensitivity: Lambda\_alpha\_E



Sensitivity: Lambda\_beta\_E

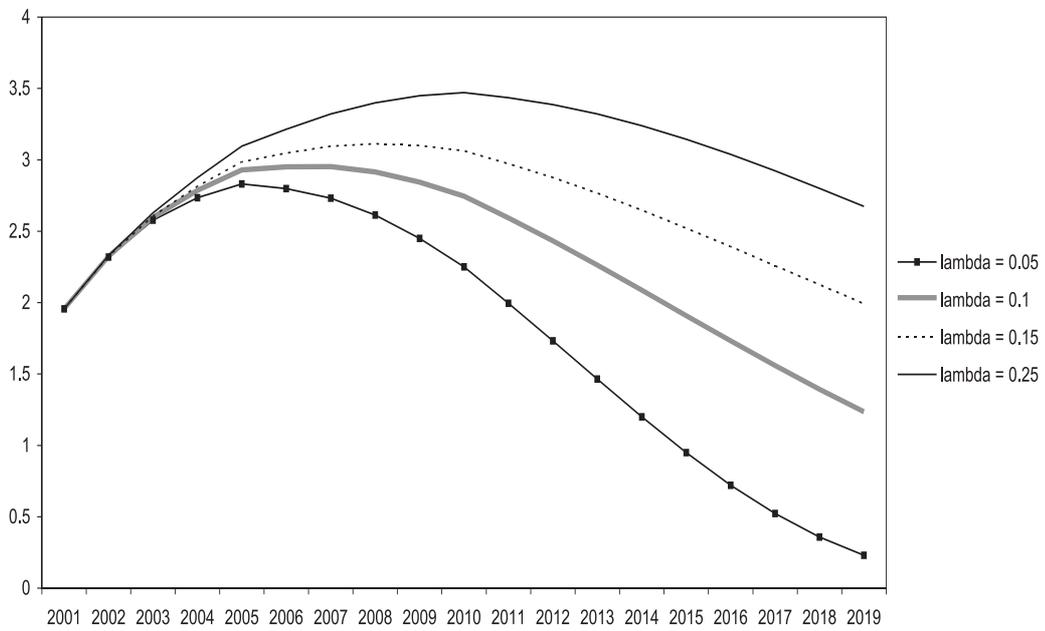
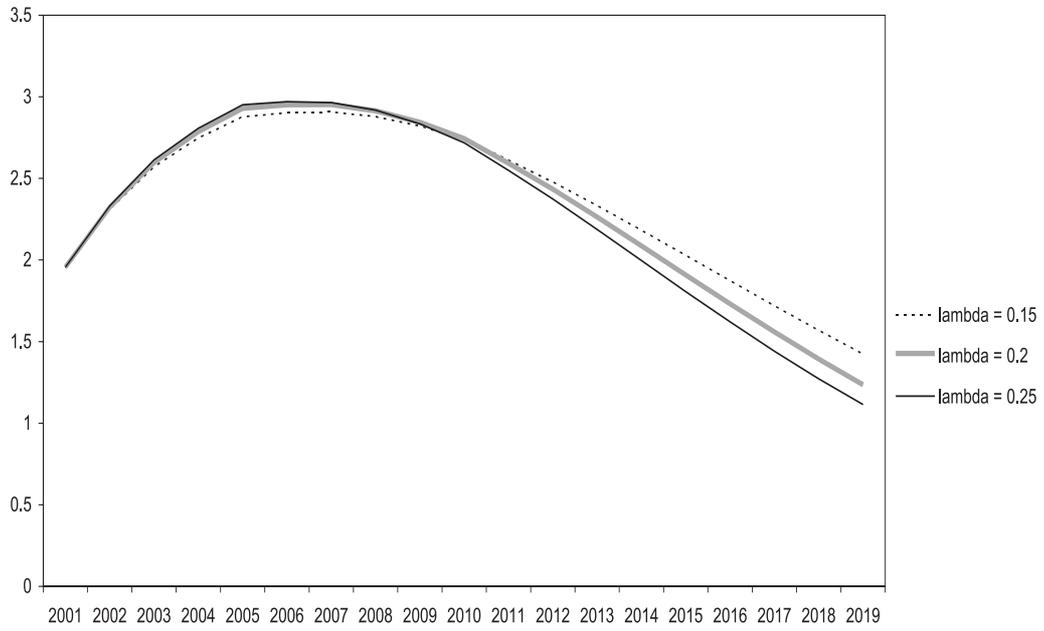


Figure 7: Sensitivities for  $\lambda_\epsilon^\alpha$  and  $\lambda_\epsilon^\beta$ .

Sensitivity: Lambda\_gamma\_E



Sensitivity: Lambda\_delta\_E

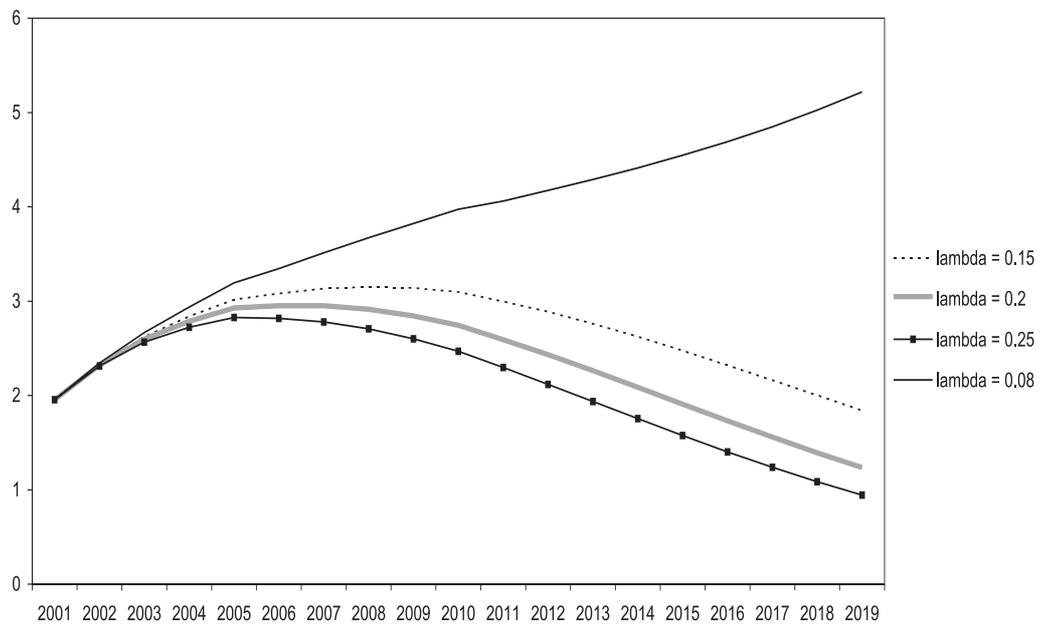


Figure 8: Sensitivities for  $\lambda_\epsilon^\gamma$  and  $\lambda_\epsilon^\delta$ .

a peak. This may be understood as a reflection on Bulgarian competitiveness: if the Bulgarian economy is slow to become competitive in the goods and services sectors, the current account may have difficulty recovering even in the medium term. According to this model, the critical value for  $\lambda^\delta$  is approximately 0.1. (This is the case whether euro and dollar parameters are considered individually or together). Thus we can say that if the elasticity of import growth to output growth does not fall by *at least 10% per annum*, then a sustainable current account will be quite unachievable.

## 6 Discussion

The model developed above is admittedly simple and the results reported should respectively be taken with a grain of salt. Nonetheless, several messages emerge from our analysis. First, it is obvious that the current situation of the Bulgarian economy, if preserved in the future, cannot sustain a declining current account deficit under a fixed exchange rate regime. While the exploding current account deficit we obtain is more of a peculiarity of the model rather than a real danger, it points to the danger of running persistent current account deficits and ultimately threatening the stability of the currency board arrangement if the present structure of the economy remains unaltered. On the other hand, even with a floating exchange rate arrangement, under the circumstances it could prove difficult to improve the external position of the economy without resorting to persistent devaluations. Even though the characteristics of the Bulgarian economy are not uncommon for transition economies and hence are not an immediate source of concern, the results underscore the fact that a change in the structure of the economy is required to bring it to a sustainable external equilibrium.

Furthermore, if we interpret the imparted trend in the elasticities in equation (21) as a proxy for changes in competitiveness (the more competitive an economy, the less sensitive it is to changes in external demand conditions), the model can be used to track the competitive position of the economy by comparing estimated elasticities with the elasticities required for a particular outcome to materialize. In the context of this modification of the model, our analysis shows that even with increasing competitiveness it may take a while before the trend of growing current account deficits is reversed. The other scenario would be to adopt a policy of gradual devaluation to bring the deficit down in the following years. The last variant of course is not considered a viable policy option at this stage.

Finally, although this work does not have a marked policy orientation, we believe that it has certain implicit policy consequences in the sense that it once again has emphasized the need to step up economic reforms in order to create conditions for higher growth and increased competitiveness.

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