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SOME TESTS OF RANDOM WALK HYPOTHESIS FOR BULGARIAN FOREIGN EXCHANGE RATES

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Contents

INTRODUCTION

A whole class of international economics and finance literature suggests that exchange rates follow a random walk pattern (for example, see Mussa (1979, p.10) and Cornell and Dietrich (1978, p.117)), or, in other words that exchange rates can be modelled by random walk just as by any other model of exchange rate determination (Meese and Rogoff (1983, p.3)). Almost all researchers use the exchange rates between major international currencies on well-established financial markets to test this hypothesis. It is interesting to check how well this hypothesis holds in a newly emerged, relatively thin foreign exchange market, operating in a high-inflation environment, like the Bulgarian one. The findings of this paper and of another author's unpublished research cast doubts on random walk presence in Bulgarian exchange rates against major international currencies. It turns out that the series of daily returns (first differences of log exchange rate levels) are stationary but correlated and therefore can be modelled better by higher-order ARIMA processes than by random walk.

The importance of random walk tests stems from their implications for forecasting exchange rates movements. If exchange rates follow random walks, this would have serious implications for businesses, economic policy makers and speculators. First, it would prove that exchange rate movements are unpredictable by structural models and the only appropriate forecasting technique is ARIMA(0,d,0)*; since the forecasting error in such models builds up very rapidly, any forecasting attempt for more than a couple of periods ahead is bound to fail. As long as there is no forward foreign exchange market in Bulgaria yet, this would leave foreign-trade-related businesses in a relatively obscure situation regarding the future domestic prices of their exports and/or

* *„d" designates the difference order necessary for achieving stationarity.*

imports. The same obscurity would not allow the Central Bank to get reliable forecasts for monetary aggregates in the near future (since foreign currency denominated deposits are included in different money concepts). On the other hand, random walk exchange rates would imply some form of market efficiency; on average, any speculators' success would be due more to a pure chance rather than to any superior forecasting ability. Assuming that speculators are rational, they would tend to disappear from the anyway thin market, diminishing the trading volumes and thereby increasing the random volatility of exchange rates.

This paper is organized as follows. Section 2 provides some theoretical background for the empirical tests performed. Section 3 presents the data and their transformation. Section 4 explains some details and outlines the results from three different tests: using the standard error of autocorrelations, Box-Pierce and Ljung-Box Q-statistics and unit roots tests. Section 5 draws conclusions. ❑

DEFINITION OF RANDOM WALK

The simplest way of stating the random walk hypothesis is that price changes are entirely unpredictable and therefore previous prices do not influence the current price; the best forecast for tomorrow's price is today's price. Market participants use all available information contained in today's information set (see the explanation of efficient market hypothesis below) to efficiently forecast prices. Therefore, the influence of all relevant factors on prices is already incorporated in today's price; if no new information is expected, the expected value of tomorrow's price is, on average, equal to today's one. To avoid the so-called „Siegel paradox" (Siegel, 1972), exchange rate series are examined as first differences of natural logarithms of exchange rates. This presentation has the advantage that every observation approximates a daily return on potential investment in foreign currency (Taylor (1986, pp.12-13)). So, in terms of daily returns, the random walk hypothesis states that every daily return is independent of previous returns. This idea has undergone a long evolution. First to ever mention it, Bachelier (1900) implies that returns are independent and identically normally distributed. Fama (1965, p. 36) relaxes the assumption of normality. Granger and Morgenstern (1970, pp.71-73) discard the identical distribution requirement. Thus the random walk hypothesis in terms of exchange rate returns can be defined by constant expected returns and a zero correlation between returns for any pair of different days (Taylor (1986, pp.8-9)). In technical terms, this looks as follows:

$$H_0: E[X_t]=E[X_{t+\tau}], \ cov(X_t, X_{t+\tau})=0 \ \text{ for all } t\wedge\tau>0 \quad (1)$$

Proving that the autocorrelation between any pair of returns is not significantly different from zero would be sufficient for the empirical testing of the hypothesis, provided the daily returns series are stationary. Therefore, the task would be transformed to appropriately detect the presence or absence of autocorrelation and stationarity. Details and different ways to do so are provided in Section 4.

It is worth noting that rejecting the random walk hypothesis does not necessarily mean rejecting the efficient market hypothesis (the opposite would be true, however). The efficient market hypothesis states that, on average (in the longer run), market participants acquainted with the information available in the beginning of a period do not make more money from frequent trading than from a simple „buy-and-hold" strategy. Contingent on the information set in question, there are several forms of market efficiency: i) weak form - the information set contains only data for past prices; ii) semistrong form - the information set contains all publicly available relevant information at that point; iii) strong form - market participants know and process all relevant information (public and private). The semistrong form is the broadly accepted version it is what is generally meant by „efficient market hypothesis" (Jensen, 1978, pp.96-97). However, transaction costs may make the profits from exploiting some statistical dependence disappear and thus the random walk hypothesis may be false while the market is efficient in some form or another (Taylor, 1986, p.134). So the absence of random walk is a necessary, but not sufficient, condition for an efficient market; on the contrary, random walk's presence is sufficient to ensure market efficiency. ❑

DATA

The exchange rates used are the Bulgarian National Bank central exchange rates expressed in Bulgarian currency (leva, BGL) per unit of foreign currency, respectively USD (the U.S. dollar), DEM (German mark), CHF (Swiss franc) and ECU (European Currency Unit). More than 95% of the trade in the Bulgarian foreign exchange market is carried out in the first two currencies (Banker newspaper, 04/12/1993); the other two are chosen as representatives for the trade in European currencies other than the German mark. Two major reasons justify their use instead of purely market exchange rates: (i) there is no unique market quotes for BGL in terms of other currencies; instead, every market player determines its own bid-ask spread; (ii) these spreads are almost evenly spread around the central exchange rates within the 1-1.5% range; thus the latter is a reasonably close approximation of the market rates. The central exchange rate is calculated as a weighted average of all the transactions performed by the commercial banks with full license for foreign exchange trade (16 major banks) the day before.The time span for all currencies is 03/12/1991 - 01/12/1993. As noted above, the daily returns (first differences in log levels) are used. ❑

TEST PROCEDURES AND RESULTS

The tests used by researchers in that field can be classified by using autocorrelation, runs and spectral statistics. Autocorrelation test statistics occupy a central place in this paper, partly because the data series are not sufficiently long for a reliable implementation of spectral analysis (Cooper (1982, p.520)). Some runs tests were also performed, but the results are contingent on a relatively strong assumption about the zero-change threshold, which limits their significance. So their results are not shown here; nevertheless, they also reject random walk.

A Simple Autocorrelation Test

The simplest autocorrelation test uses the following statistic:

$$S = \sqrt{n} * r_{(i)} \qquad (2)$$

This statistic tests for autocorrelation between daily returns. Here „n" stands for the number of observations and „r(i)" is the i-th autocorrelation coefficient. „S" is expected to be approximately N(0,1), when the null hypothesis (random walk) is true. Therefore, the null is rejected when $S > |1.96|$. The basic advantage of this test is its simplicity and computational easiness; however, since 1/n is a lower-bound estimate of the autocorrelation variance, it tends to be biased in favor of the null hypothesis. Thus, when neither autocorrelation coefficient is significantly different from 0, this is a sure indication of random walk; however, it is possible for some autocorrelation coefficients to be significant and the null still not to be rejected. Therefore, this test rejects the null hypothesis too often.

The results are shown in Table 1. For each series, 30 autocorrelation and partial autocorrelation coefficients are computed. A coefficient is significant (at 95% confidence level) if it exceeds in magnitude 2/square root of „n“.

Table 1: Number of significant autocorrelation coefficients in each of the series

	ac	pac
BGL/USD	**10**	**7**
BGL/DEM	**10**	**6**
BGL/CHF	**7**	**5**
BGL/ECU	**10**	**5**

The results clearly show that the presence of auto-correlation cannot be rejected - too many autocorrelation and partial autocorrelation coefficients are significantly different from zero. The first and second autocorrelation coefficients(not shown here) are significant* in each series. Therefore, this test rejects the random walk hypothesis.

Box-Pierce and Ljung-Box Q-statistics.

Box and Pierce (1970, pp.1509-1526) define the following statistic in testing for autocorrelation:

$$Q_{BP}=n_* \Sigma_{j=1}^{k}(r_1)^2 \qquad (3)$$

** Except for BGL/CHF, where the first autocorrelation coefficient is barely insignificant.*

Here the null is that all autocorrelations are not significantly different from zero, i.e. the series tested is white noise, which, in a first-difference series representation, is equivalent to random walk. Under the null hypothesis, Q has a chi-squared distribution with „k“ degrees of freedom, where „k“ is the number of lags*.

This statistic, however, has a low power even for large samples - it produces significant values too rarely. Ljung and Box (1978) define another statistic:

$$Q_{LB}=n_{*}(n+2)\sum_{j=1}^{k}\frac{r_j^2}{n-j} \qquad (4)$$

It has the same large-sample property distribution, but produces higher values than Box-Pierce Q in small samples. Furthermore, its significance levels are closer to those given by asymptotic theory (Mills, (1992), p.145).

The results are shown in Table 2. With the exception of Q(1) for BGL/CHF exchange rate series, all Q-statistics are significant at a 5% significance level, thereby rejecting the null with less than 5% probability for error. Note that trought the series the big jump is produced by the second autocorrelation coefficient, implying a highly significant correlation between returns separated by two days. Although this phenomenon does not have any meaningful explanation, it does suggest that an AR(2) or MA(2) process could model the series better than the random walk.

Summing up, both Q-statistics imply some kind of autocorrelation between the daily returns at all possible lags, thereby casting more and more doubts on the random walk hypothesis.

* *If the series has been subject to ARIMA (p,d,q) analysis, the degrees of freedom are k-p-q; if, in addition, a constant is used, they are k-p-q-1.*

Table 2: Box-Pierce and Ljung-Box statistics for each of the series

	BGL/USD		BGL/DEM		BGL/CHF		BGL/ECU		CHI(k)
	Q(BP)	Q(LB)	Q(BP)	Q(LB)	Q(BP)	Q(LB)	Q(BP)	Q(LB)	(95%)
1	4.98	5.01	5.99	6.03	3.63	3.66	6.75	6.80	3.84
2	34.29	34.57	30.80	31.05	24.91	25.12	33.32	33.59	5.99
3	35.32	35.62	31.17	31.42	25.14	25.34	33.51	33.78	7.81
4	57.82	58.40	47.22	47.68	44.46	44.92	51.89	52.40	9.49
5	58.99	59.59	47.31	47.77	44.63	45.09	51.94	52.45	11.10
6	59.38	59.99	47.38	47.84	44.90	45.37	51.95	52.46	12.60
7	61.31	61.95	48.60	49.09	45.73	46.21	53.87	54.42	14.10
8	61.49	62.14	48.82	49.32	45.92	46.40	54.26	54.82	15.50
9	63.47	64.17	49.50	50.01	46.13	46.61	55.44	56.02	16.90
10	72.80	73.74	56.14	56.83	54.93	55.65	61.42	62.17	18.31
11	81.35	82.53	60.38	61.18	58.73	59.55	66.11	66.99	19.70
12	95.71	97.33	66.91	67.91	62.44	63.38	72.65	73.72	21.00
13	102.58	104.43	68.22	69.27	63.23	64.20	74.45	75.58	22.40
14	110.87	113.01	76.40	77.73	72.42	73.71	83.25	84.69	23.70
15	112.14	114.33	78.38	79.79	74.99	76.38	84.83	86.33	25.00
16	123.12	125.74	88.10	89.90	82.09	83.76	94.15	96.03	26.30
17	125.55	128.28	93.57	95.60	87.16	89.04	99.72	101.83	27.60
18	132.08	135.10	99.78	102.07	92.43	94.54	106.04	108.42	28.90
19	132.59	135.63	99.81	102.11	92.44	94.55	106.05	108.44	30.10
20	132.59	135.63	99.84	102.15	92.45	94.56	106.08	108.47	31.40
21	134.83	137.98	101.37	103.75	93.97	96.17	107.12	109.56	32.70
22	136.57	139.82	102.49	104.93	94.84	97.08	108.29	110.80	33.90
23	138.61	141.98	103.00	105.47	95.29	97.56	109.16	111.71	35.20
24	139.19	142.59	103.72	106.23	96.12	98.43	109.73	112.32	36.40
25	142.12	145.69	106.87	109.57	99.59	102.11	113.36	116.17	37.70
26	142.79	146.41	107.62	110.37	100.90	103.51	114.15	117.01	38.90
27	143.44	147.10	107.69	110.44	100.95	103.56	114.36	117.23	40.10
28	143.50	147.17	107.79	110.56	101.01	103.62	114.45	117.33	41.30
29	144.97	148.74	108.04	110.82	101.52	104.17	114.68	117.57	42.60
30	145.09	148.87	109.31	112.18	102.43	105.14	115.59	118.55	43.77

Chi-squared table values are from Wonnacut and Wonnacut (1979, p. 540)

Unit Roots Tests.

The results presented in sections 4.1. and 4.2. may be challenged by the following argument: maybe they reject the random walk hypothesis simply because the daily returns series are nonstationary; first-order differencing of log levels might prove insufficient to induce stationarity. If so, it is possible for the exchange rates to be random walks with drifts, a plausible hypothesis in a macroeconomic environment characterized by high inflation (over 80% for 1992); in that case the daily returns would be correlated because they include a common drift. So a powerful test is needed to test for stationarity. Such test is the Augmented Dickey-Fuller test for unit roots.

The theoretical framework of this test in its different forms is presented in Fuller (1976) and Dickey and Fuller (1979); a useful „cookbook" for applied researchers is Perman (1991). Summarized, the Augmented Dickey-Fuller (ADF) test involves running the following regression:

$$\Delta Y_t = \alpha_0 + \alpha_1 * T + \rho * Y_{t-1} + \Sigma_{i=1}^{m} \beta_i * \Delta Y_{t-i} + e_t$$

where Y is the tested variable and T is a deterministic time trend. Including a constant and a time trend is optional and is based on a preliminary analysis regarding the nature of the stochastic process explored. However, it is always better to include these variables even if they are irrelevant, than to omit them if chances are for them to be relevant. Anyway, the test is safely conclusive if all three specification options (without a constant and a time trend, with a constant only, with both the constant and time trend) produce similar results.

The coefficient of major importance in this regression is „rho". Under the null hypothesis (random walk) „rho" is zero. If the alternative hypothesis is true, „rho" is negative*. Therefore, the task is to test whether „rho" would be significantly different from zero. Fuller (1976) has proved that under the null hypothesis of random walk („rho"=0) standard t- and F-distributions are not defined. So a conventional t-test cannot be used. In the same book Fuller also developed the so-called „unit root distribution" and derived critical values for selected sample sizes. Later, MacKinnon (1990) generalized Fuller's critical values for any sample size. So the computed t-statistic for „rho" has to be compared to Fuller (Mackinnon) critical values for selected sample size, specification option and level of significance. Since both numbers are (usually) negative, the null is rejected if the computed t-value is less than the critical value.

The residuals from the above regression are assumed to be white noise. In order to ensure this assumption, lagged difference terms are added until white noise residuals are achieved. The latter are tested by Ljung-Box or Box-Pierce Q-statistics (see p. 8), as well as by the Lagrange Multiplier (LM) test. The latter is superior to the former two in detecting a low-order serial correlation (Mills, 1992, p. 145-150).

The results are presented in Table 3. A preliminary analysis has suggested that neither a constant, nor a time trend should be included in the test equations; however, in order to avoid the possibility for misspecification which would affect the results, all three options have been tested. As expected, the results are almost the same. Technical details are given below Table 3 and proofs that white noise residuals are achieved are presented in Table 4.

** Of course, it is possible for „rho" to be positive. However, this would imply an „explosive" stochastic process, which is implausible as far as economic variables are concerned.*

Table 3: Unit Root Results and Trend Statistics

	Augmented Dickey-Fuller Tests			t-tests for
Series	no constant no trend	with constant no trend	with constant with trend	deterministic trends
BGL/USD	-7.92	-8.07	-8.09	-0.69
(10 lags)	-0.76	-0.78	-0.79	
BGL/DEM	-7.54	-7.60	-7.60	-0.41
(4 lags)	-0.58	-0.59	-0.59	
BGL/CHF	-7.79	-7.82	-7.81	-0.21
(5 lags)	-0.64	-0.65	-0.65	
BGL/ECU	-7.49	-7.53	-7.54	-0.46
(4 lags)	-0.57	-0.58	-0.58	
	MacKinnon critical values:			Student
	no constant no trend	with constant no trend	with constant with trend	critical t-values:
1%	-2.57	-3.45	-3.98	2.58
5%	-1.94	-2.87	-3.42	1.96
10%	-1.62	-2.57	-3.13	1.65

Data are daily from 03/12/1991 to 01/12/1993. Augmented Dickey-Fuller t-statistics and coefficients of the lagged variable are reported in the first and second line respectively, for each exchange rate series. Each regression uses a different number of lagged differences (shown in parentheses) to achieve white noise residuals. The 1%, 5% and 10% critical values are taken from MacKinnon (1990) and represent a mere extension of the original Fuller table (Fuller (1976), Table 8.5.2) for any size of the sample and for each of the three computing options presented.

The t-tests for deterministic trends are obtained by regressing the series on the respective number of lagged difference terms, the constant and the time trend. The Student critical t-values are from Wonnacut and Wonnacut, (1979, p.539).

Table 4: Residual Check

Series and statistics	no constant no trend	with constant no trend	with constant with trend
BGL/USD			
Ljung-Box Q-stat(22)	30.39	30.36	30.72
Ljung-Box Q-stat(6)	1.39	1.20	1.25
Breusch-Godfrey LM-stat(6)	0.96	1.12	1.16
BGL/DEM			
Ljung-Box Q-stat(22)	32.59	32.94	33.30
Ljung-Box Q-stat(6)	4.33	4.36	4.40
Breusch-Godfrey LM-stat(6)	3.89	5.17	5.20
BGL/CHF			
Ljung-Box Q-stat(22)	30.64	30.85	31.00
Ljung-Box Q-stat(6)	3.51	3.53	3.55
Breusch-Godfrey LM-stat(6)	3.13	4.13	4.15
BGL/ECU			
Ljung-Box Q-stat(22)	31.90	32.20	32.65
Ljung-Box Q-stat(6)	4.02	4.06	4.10
Breusch-Godfrey LM-stat(6)	3.78	4.86	4.88

Chi-squared table values:

	22 d.f.	6 d.f.
1%	40.30	16.80
5%	33.90	12.60
10%	30.80	10.60

The Ljung-Box Q-statistic is used instead of the Box-Pierce Q-statistic because of its better properties (see p.8). The figure in parentheses shows the number of lags used. The 22 lags are square root on number of observations; recommended by Poskitt and Tremayne (1981). The Breusch-Godfrey LM statistics is explained in detail in Johnston (1984, pp.319-321) . The Chi-squared table values are from Wonnacut and Wonnacut, (1979, p.540)

As an additional check on the sensitivity of Dickey-Fuller t-statistics to the number of lagged differences included, augmented Dickey-Fuller tests are performed for each series with plus-minus one and two lags from the optimal number of lags, given in Table 3. These tests are shown in Table 5. Note that the Dickey-Fuller t-statistics are either quite close to these in Table 3, or, if different, are of a greater significance (cf. BGL/DEM and BGL/ECU results). So the null hypothesis (unit root) is definitely rejected anyway.

Table 5: Augmented Dickey-Fuller Tests for Lower- and Higher-Order Lagged Differences.

Number of lagged differences for each series	Option		
	no constant no trend	with constant no trend	with constant with trend
BGL/USD			
8	-7.26	-7.39	-7.43
9	-7.63	-7.74	-7.75
11	-8.21	-8.38	-8.42
12	-7.91	-8.10	-8.15
BGL/DEM			
2	-11.23	-11.30	-11.30
3	-8.72	-8.79	-8.81
5	-7.79	-7.83	-7.82
6	-7.71	-7.79	-7.80
BGL/CHF			
3	-8.57	-8.63	-8.65
4	-7.43	-7.47	-7.47
6	-7.67	-7.72	-7.73
7	-7.38	-7.45	-7.47
BGL/ECU			
2	-11.12	-11.17	-11.19
3	-8.55	-8.62	-8.64
5	-7.71	-7.75	-7.74
6	-7.77	-7.83	-7.84

All t-statistics (in both Table 3 and Table 5) are significant at the 1% significance level, thereby strongly implying stationarity in each of the series. Furthermore, when the test equation includes a time trend and/or a constant, they both turn to be highly insignificant (indirectly proven by the similarity of the results under different options). So this test rejects the presence of both the deterministic and stochastic trend in daily return series, as well as the possibility of a drift. Therefore, the series in question are definitely stationary, yet correlated - so they are not random walks. ❑

CONCLUSION

The random walk hypothesis tested in this paper states that the daily returns series of exchange rates should be stationary and uncorrelated. The three types of tests performed here suggest that they are certainly stationary, yet correlated. While scientific perfection requires some other tests to be performed (for example, for conditional variance), the research done so far clearly indicates that an ARIMA model of higher order (probably of order 2 in AR and/or MA terms) could model Bulgarian exchange rate series better than the random walk model. Subject to further research is to check if this conclusion holds under a different time span, as well as to determine the best model (autoregressive, structural or perhaps mixed) to forecast the daily exchange rates of Bulgarian currency against the major international currencies. ❑

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